The Inverted Pendulum Experiment

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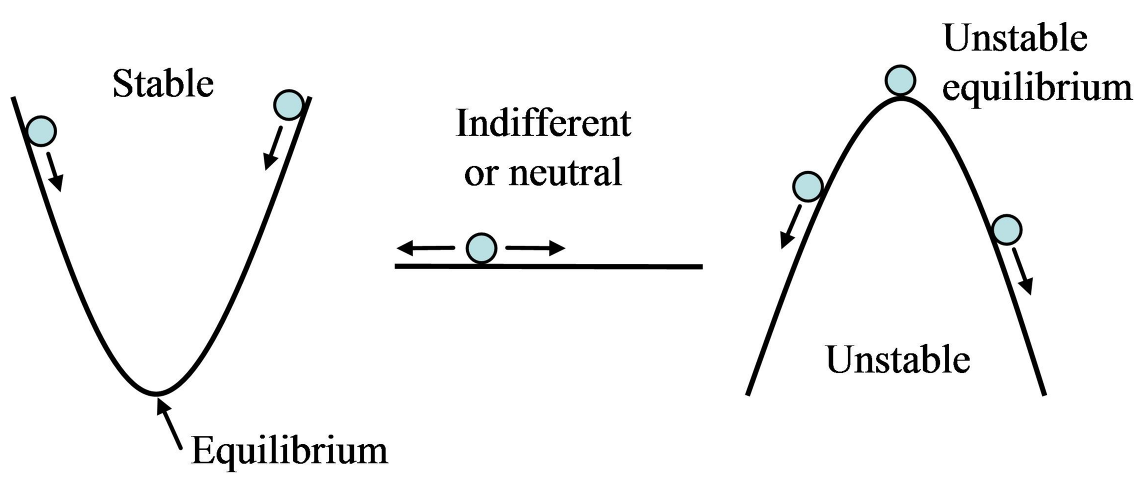
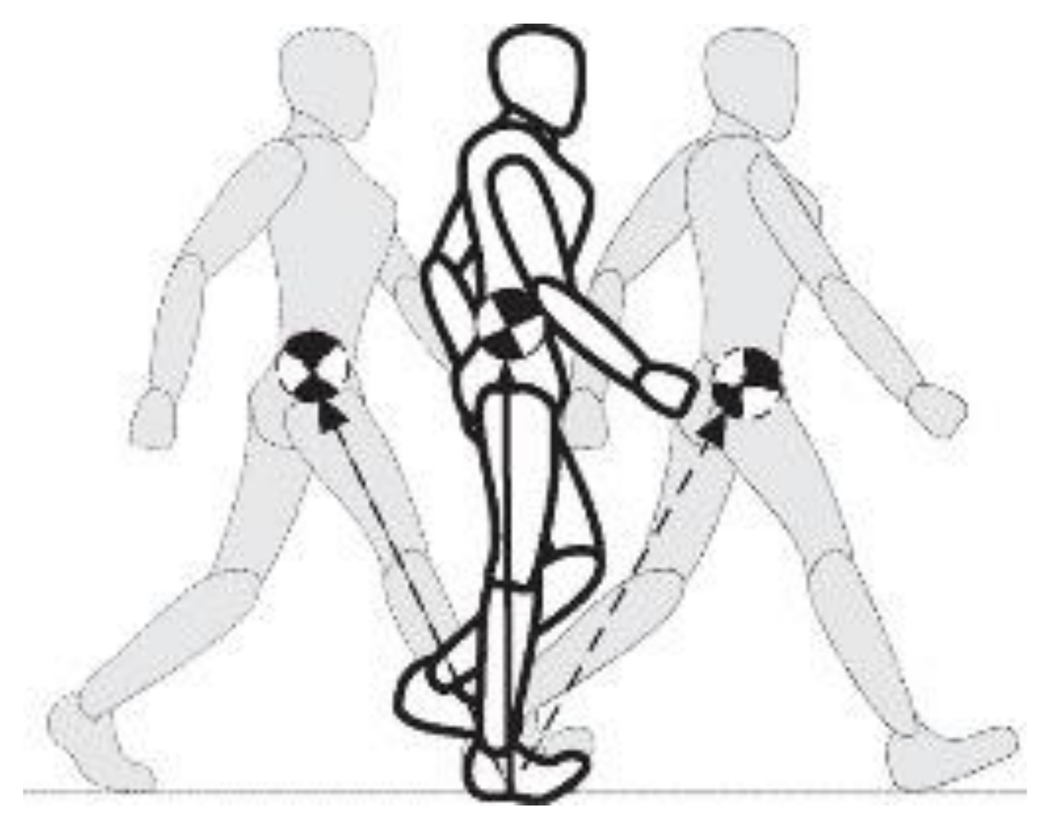


Stability and Equilibrium



Human Biomechanics and the Inverted Pendulum

Computer-Controlled Exoskeleton

Abstract

In this experiment, we examine the non-linear dynamics of a mechanical system consisting of an inverted pendulum with one free-turning rotational degree-of-freedom attached to a computer-controlled cart with one linear degree-of-freedom. Using a Quanser Linear Servo Base Unit with Inverted Pendulum and paired software package, we used first principles to develop the non-linear control system needed to move the pendulum from stable equilibrium to unstable equilibrium and maintain unstable equilibrium. This combines the self-erecting inverted pendulum experiment and the classic pendulum experiment. Through the paired software package, we were able to derive the dynamic equations to develop the transfer function and proportional-velocity feedback system that describe the linear motion of the cart, successfully creating the non-linear control system for both phases of the experiment.

Background

- Classic robotics control system problem.
- Non-linear system.
- Number of degrees-of-freedom are greater than the number of controlled variables.
- System must be linearized and evaluated in state space to mange small deviations from unstable equilibrium.

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Using a motorized cart on a straight track, we did a lot of math and developed a program to control the cart in order to balance an upside-down pendulum.

Experimental Procedures

- Mathematical analysis from first principals.
- Linearization of equations of motion.
- State-space evaluation.
- Design PID controller to stabilize pendulum.

Analysis

Summation of Forces

$$\sum F_{x,c} = m_c\ddot{x}_c = F - R_x$$

$$\sum F_{x,p} = m_p\ddot{x}_p = R_x$$

$$\sum F_{y,p} = m_p\ddot{y}_p = -R_y - m_p g$$

$$\sum M_{0,p} = I\ddot{\theta} = R_x L cos\theta - R_y L sin\theta$$

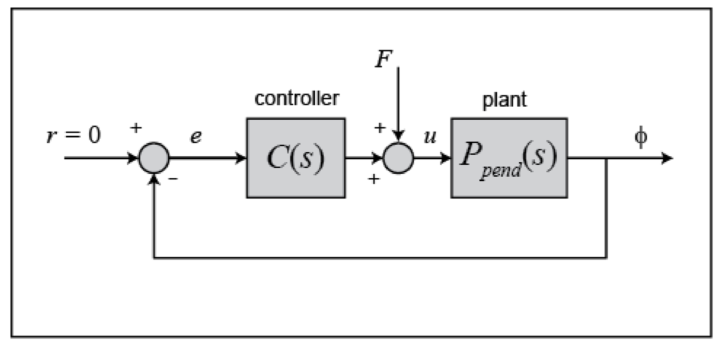
Equations of Motion

$$\begin{bmatrix} I + m_pL^2 & -m_pLcos\theta \\ -m_pLcos\theta & m_c + m_p \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{x}_c \end{bmatrix} = \begin{bmatrix} m_pgLsin\theta \\ -m_pLsin\theta\dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} 0 \\ F \end{bmatrix}$$

Linearized Equations of Motion

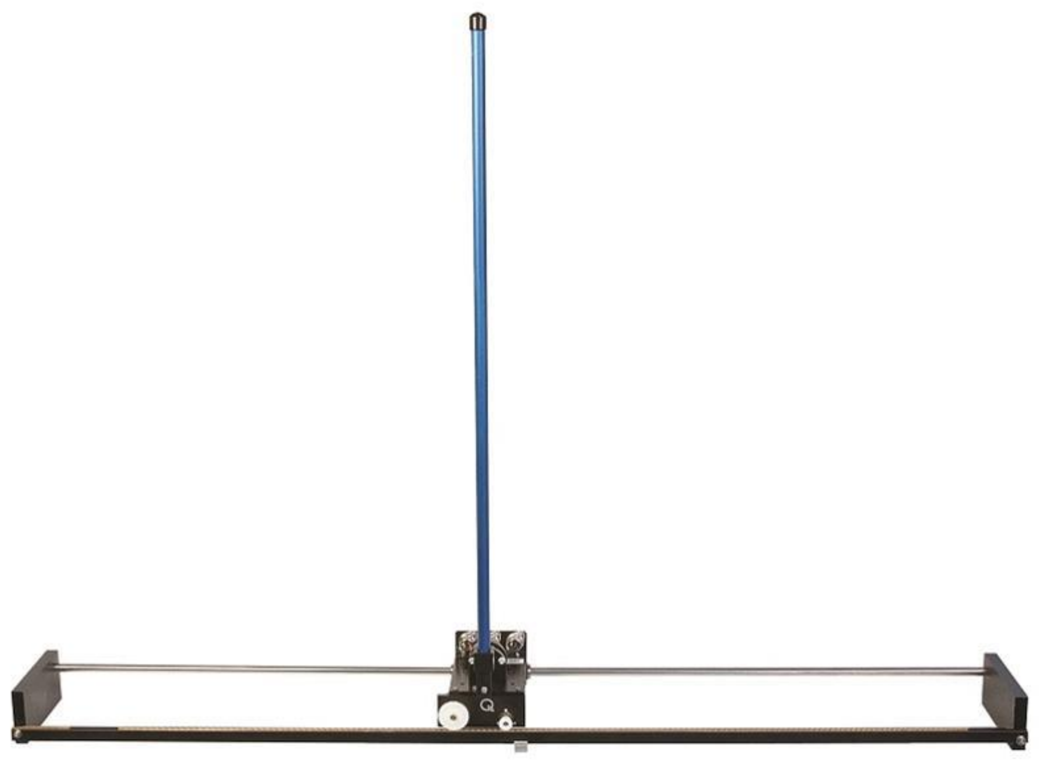
$$\begin{bmatrix} I + m_pL^2 & m_pL \\ m_pL & m_c + m_p \end{bmatrix}\begin{bmatrix} \ddot{\phi} \\ \ddot{x}_c \end{bmatrix} = \begin{bmatrix} -m_pgL\phi \\ u \end{bmatrix}$$

PID Controller Design

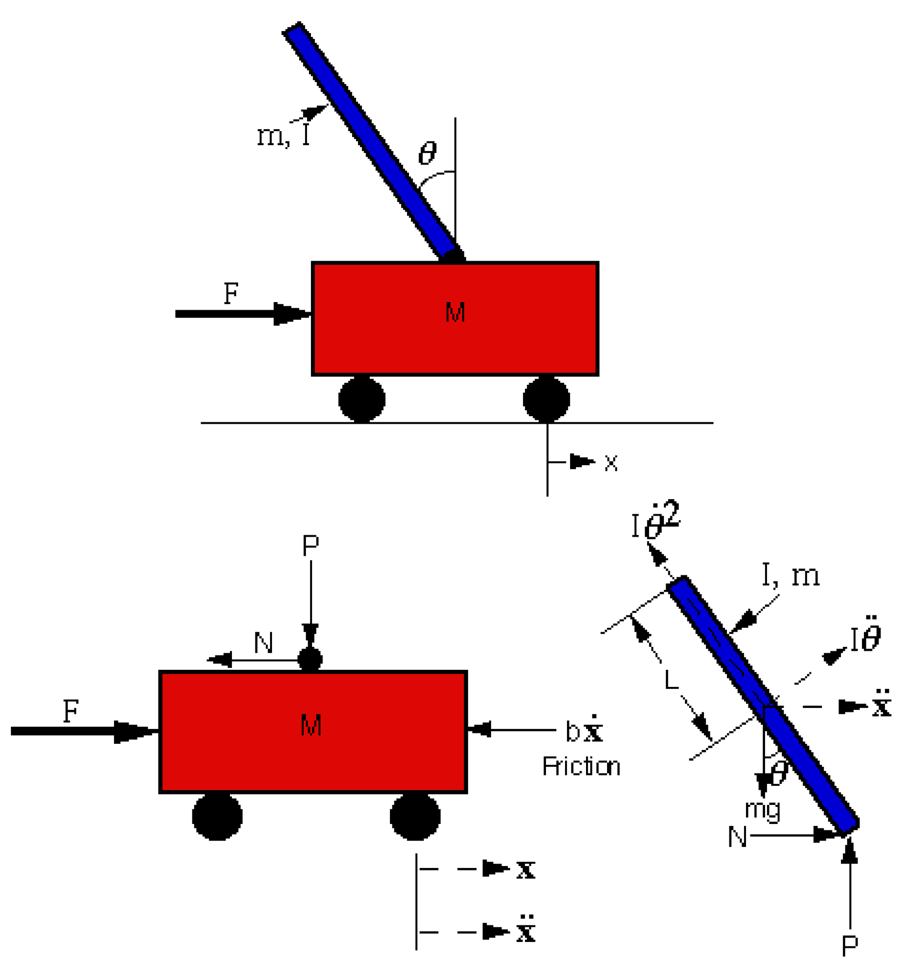


Conclusions

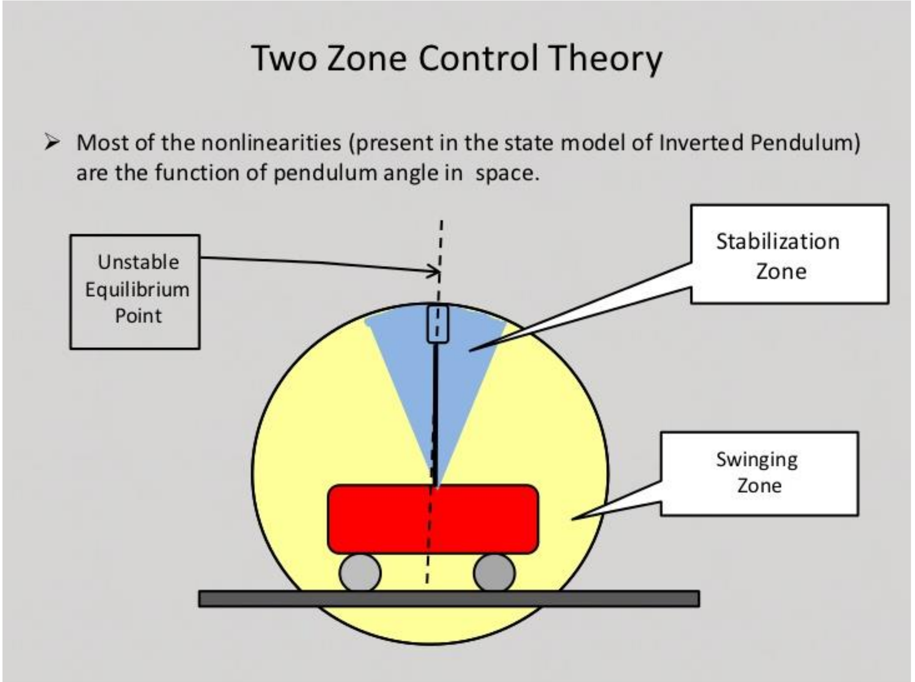
An inverted pendulum can be controlled by the linear motion of a cart by a linear controller in a small region of operation. Future work includes the development of a proportional-integral-derivative controller to stabilize the system as well as move the system from stable equilibrium to unstable equilibrium.



Quanser Inverted Pendulum



Free Body Diagram of Pendulum & Cart



Stabilization Zone

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