Spatiotemporal Analysis of Respiratory Tract Infection Dynamics

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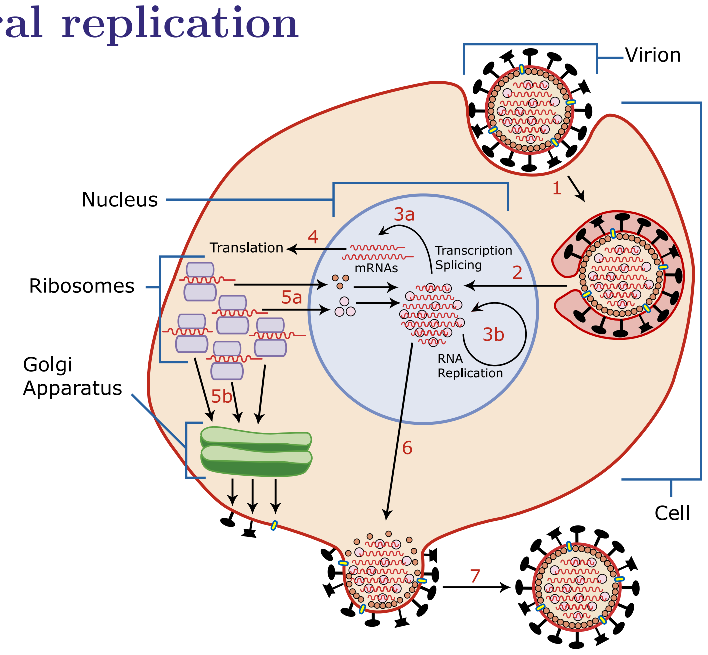
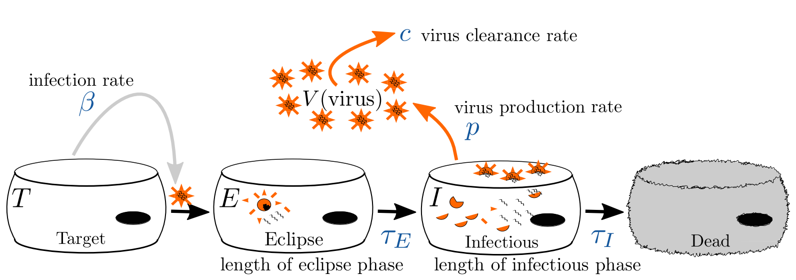
Background

- Many respiratory viral infections exhibit both uncomplicated and severe clinical illness.
- Uncomplicated infections are localized near the top of the respiratory tract, while severe infections spread deeper into the lower respiratory tract.
- We construct a continuous spatial model to study spread of virus down the respiratory tract via mucous transfer speed, or advection.

Viral replication



Mathematical model

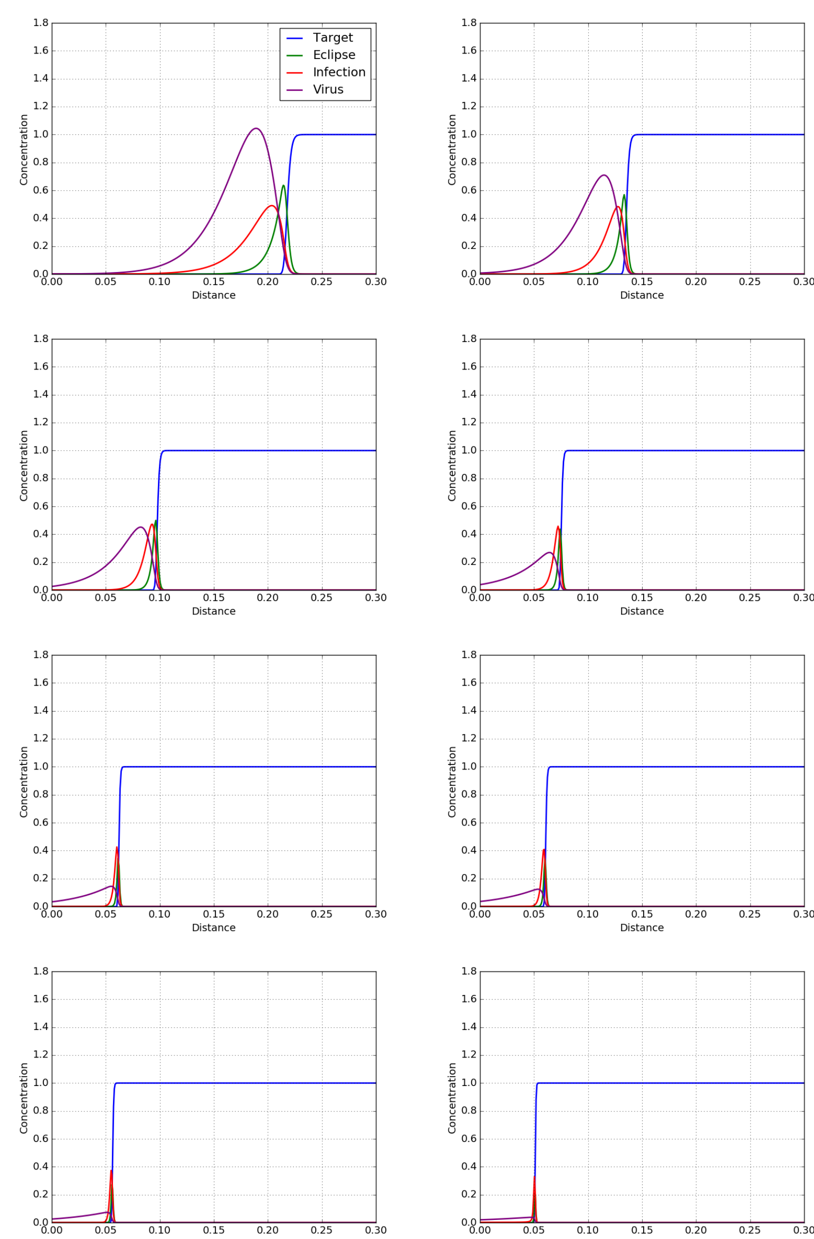


$$
\begin{aligned}
\partial_t T &= -\beta T V \\
\partial_t E &= \beta T V - \frac{E}{\tau_E} \\
\partial_t I &= \frac{E}{\tau_E} - \frac{I}{\tau_I} \\
\partial_t V &= pI - cV + D\partial_x^2 V + \nu \partial_x V
\end{aligned}
$$

- Across space, virus replicates by infecting healthy target cells.
- Model includes two transport mechanisms: diffusion and advection.

Results

We implemented the model using custom-written Python code. Results shown below are snapshots of target cells, eclipse cells, infectious cells, and virus in a one-dimensional respiratory tract.



- We see that as advection increases, wave speed decreases.
- Sufficiently large advection values keep virus from traveling down the respiratory tract.
- As advection increases, the peak viral load decreases.
- Similar results appear when decreasing the value for diffusion.

Analysis

The solution to the system is a wave which satisfies the conservation equation:

$$\partial_t f(x,t) - \alpha \partial_x f(x,t) = 0$$

This is used to express the system in terms of the moving coordinate frame $x = x_o + \alpha t$. Then, we take laplace transforms over the space domain and arrive at the convolution equation:

$$V = \frac{p}{k\tau_E D}\left[e^{-\frac{1}{\alpha\tau_E}x} * e^{-\frac{1}{\alpha\tau_I}x} * e^{-\frac{(\nu-\alpha)}{2D}} \sinh kx * T'\right]$$

The convolution integral is supported on the interval $0 \le x < \infty$, considering the domain of the laplace transform.

$$f(x) * g(x) = \int_0^x f(\xi)\, g(x-\xi)\, d\xi$$

And noting that functions of physical concentration are absolutely differentiable, we may obtain an expression for $V(x)$ in terms of $T(x)$:

$$V = C(x) * T$$

- The steady wave solution of V is expressed as a convolution of $T(x)$ and a known function $C(x)$
- Expansion of terms in the convolution allows for simple numerical waveform approximations

Conclusions

- Our model shows distinct behaviors as advection and diffusion rates are changed, including infections isolated to the upper respiratory tract and long-lasting severe infections.
- High advection prevents wave propagation, while lower advection can lead to deeper and stronger infections.
- Infants and the elderly are known to have lower advection rates, perhaps explaining the higher incidence of lower respiratory tract infections in these groups.

Future Directions

- Derive explicit solutions to the convolution equation.
- Look for necessary conditions on the wave speed and amplitude.
- Examine parameter space portraits for diffusion and advection.