Wound Healing Process Modeling Using Partial Differential Equations and Deep Learning

SRS 2021

Hy Dang
Advisor: Prof. Ken Richardson

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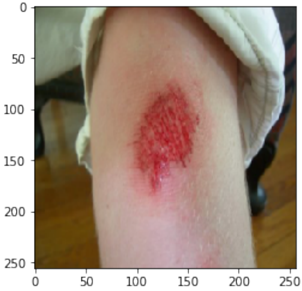
Overview

Motivation

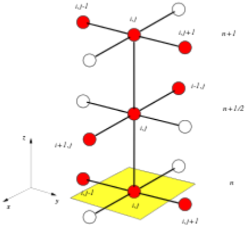


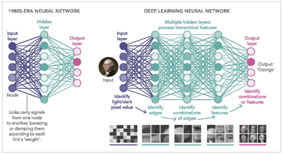
The process of successful skin healing from a wound involves different combinations of interactions. Moreover, by clearly understanding this process, we can provide and determine the appropriate amount and type of medicine to give to patients with varying types of wounds. With a good mathematical model, researchers can more quickly determine the effectiveness of healing therapies.

→ Thus, this can improve the healing process of patients.

Overview

Concepts - Methods





- In this research, we use a modified Alternating Direction Implicit (ADI) method to solve a partial differential equation that models the wound healing process.
- Wound images are used as our dataset experiment. To segment the image's wound, we implement U-Net, a deep neural network model, as our model for this segmentation problem.
- $\rightarrow$ We believe the combination of modified-ADI and Deep Learning provides an excellent fit to the wound healing process.

Workflow

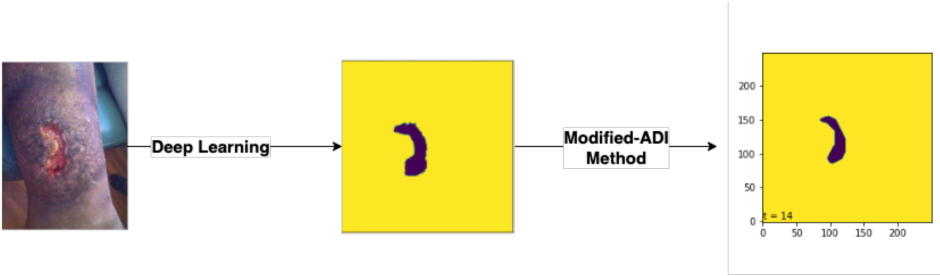


Figure: Combining all the methods

Mathematical Model

Diffusion Equation

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right); \\
(x, y) &\in (0, L) * (0, L); t > 0; \\
u &= g(x, y, t), \;\; x = [0, L]; y = [0, L] \\
u(x, y, t) &= I(x, y); \;\; x \in [0, L] * [0, L].
\end{aligned} \tag{1}
$$

We use the Diffusion Equation (Heat Equation) to model and formulate the wound healing process to deal with this problem.

Parameters

- Threshold: θ
- Speed Parameter: k
- Resolution of images: $\Delta x, \Delta y, \Delta t$

Forward-Euler Method

Forward-Euler Method

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = \frac{u_{i-1,j}^{n} - 2u_{i,j}^{n} + u_{i+1,j}^{n}}{\Delta x^2} + \frac{u_{i,j-1}^{n} - 2u_{i,j}^{n} + u_{i,j+1}^{n}}{\Delta y^2}$$

$$\iff u_{i,j}^{n+1} = \frac{\Delta t}{\Delta x^2}(u_{i-1,j}^{n} - 2u_{i,j}^{n} + u_{i+1,j}^{n}) + \frac{\Delta t}{\Delta y^2}(u_{i,j-1}^{n} - 2u_{i,j}^{n} + u_{i,j+1}^{n})$$

Alternating Direction Implicit Method

ADI Method

$$-\frac{\Delta t}{\Delta x^2}u_{i-1,j}^{n+\frac{1}{2}} + 2(1+\frac{\Delta t}{2\Delta x^2})u_{i,j}^{n+\frac{1}{2}} - \frac{\Delta t}{2\Delta x^2}u_{i+1,j}^{n+\frac{1}{2}} = 2u_{i,j}^{n} + \frac{\Delta t}{2\Delta y^2}(u_{i,j-1}^{n} - 2u_{i,j}^{n}) \tag{4}$$

Similar, we have for another sub time step:

$$-\frac{\Delta t}{\Delta y^2}u_{i,j-1}^{n+1} + 2(1+\frac{\Delta t}{2\Delta y^2})u_{i,j}^{n+1} - \frac{\Delta t}{2\Delta y^2}u_{i,j+1}^{n+1} = 2u_{i,j}^{n+\frac{1}{2}} + \frac{\Delta t}{2\Delta y^2}(u_{i,j-1}^{n+\frac{1}{2}} - 2u_{i,j}^{n+\frac{1}{2}}) \tag{5}$$

Alternating Direction Implicit Method (cnt)

- Alternating direction implicit methods are finite difference methods for solving parabolic PDEs in two and three dimensions. The convergence properties of these methods are well-understood.
- We replace a two-dimensional problem with a series of one-dimensional problems to generate a computationally efficient and **STABLE** algorithm

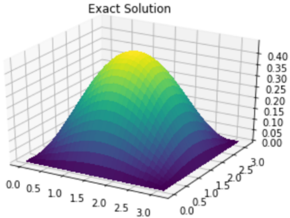


Figure 7: Exact Solution

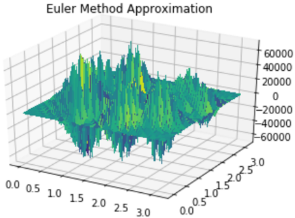


Figure 8: Euler Approximation Method

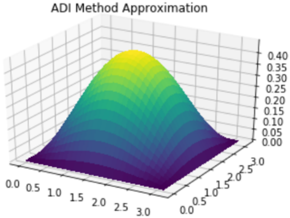


Figure 9: ADI Approximation Method

Workflow

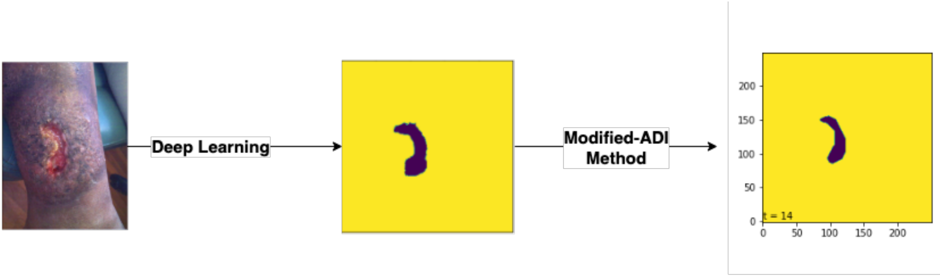


Figure: Combining all the methods

Deep Learning

- The problem is the segmentation problem, where we want to calculate the wound area accurately from the picture. Deep Learning is a possible way to do that when we can solve the computer vision problem by analyzing all the pictures that we have
- We have tried different models to consider the most suitable one for this segmentation problem. And we found that the two promising models are U-Net [RFB15] and Residual U-Net [BJJ89]

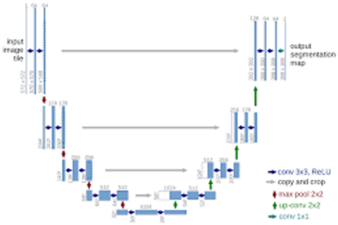


Figure: U-Net Model [RFB15]

Deep Learning Result

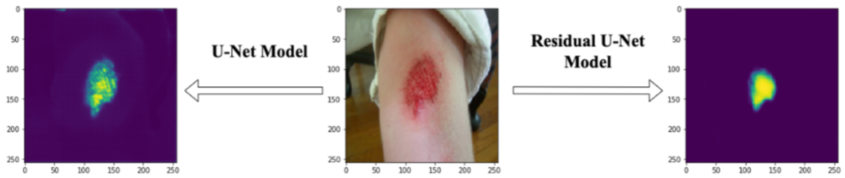


Figure: Results generated by Deep Learning models

Deep Learning Result

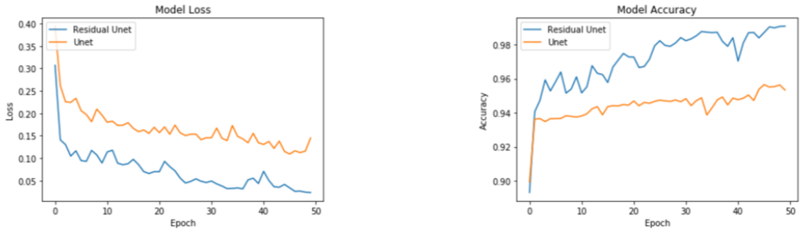


Figure 2: The loss and accuracy of U-Net and Residual U-Net models on the training set.

Workflow

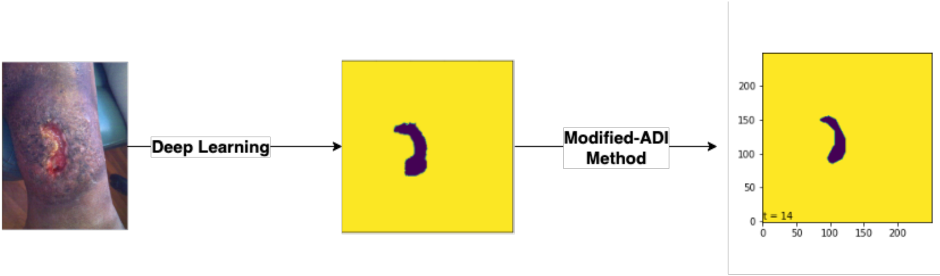


Figure: Combining all the methods

Modified-ADI Result, Predicted Results By ADI - Date 14

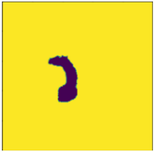
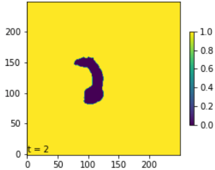
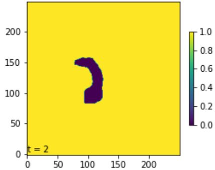


Figure: Actual Data - Date 0

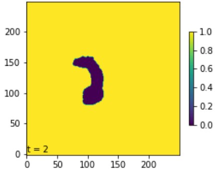
Figure: Actual Data - Date 14



$(k, \theta) = (0.6, 0.4)$



$(k, \theta) = (0.9, 0.4)$



$(k, \theta) = (0.6, 0.8)$

Overview

Result and Conclusion

- We proved the unconditional stability of ADI Method for Approximation problems and modified the method to fit the Wound Healing Model.
- We analyzed the Deep Learning models that are used to segment the wounds in the pictures.
- **Goal**: Find the best parameters for each patient and test the predictability of the model.

References

Alom, Md Zahangir, Mahmudul Hasan, Chris Yakopcic, Tarek M. Taha, and Vijayan K. Asari. "Recurrent residual convolutional neural network based on u-net (r2u-net) for medical image segmentation." arXiv preprint arXiv:1802.06955 (2018).

Barkau, Robert L., Marc C. Johnson, and Michael G. Jackson. "UNET: A model of unsteady flow through a full network of open channels." In Hydraulic Engineering, pp. 1041-1046. ASCE, 1989.

Thank you!