Distribution of Random Curves

Hoang Long (Ace) Nguyen, Advisor: Dr. Ken Richardson
Department of Mathematics, Texas Christian University, Fort Worth, TX

Abstract

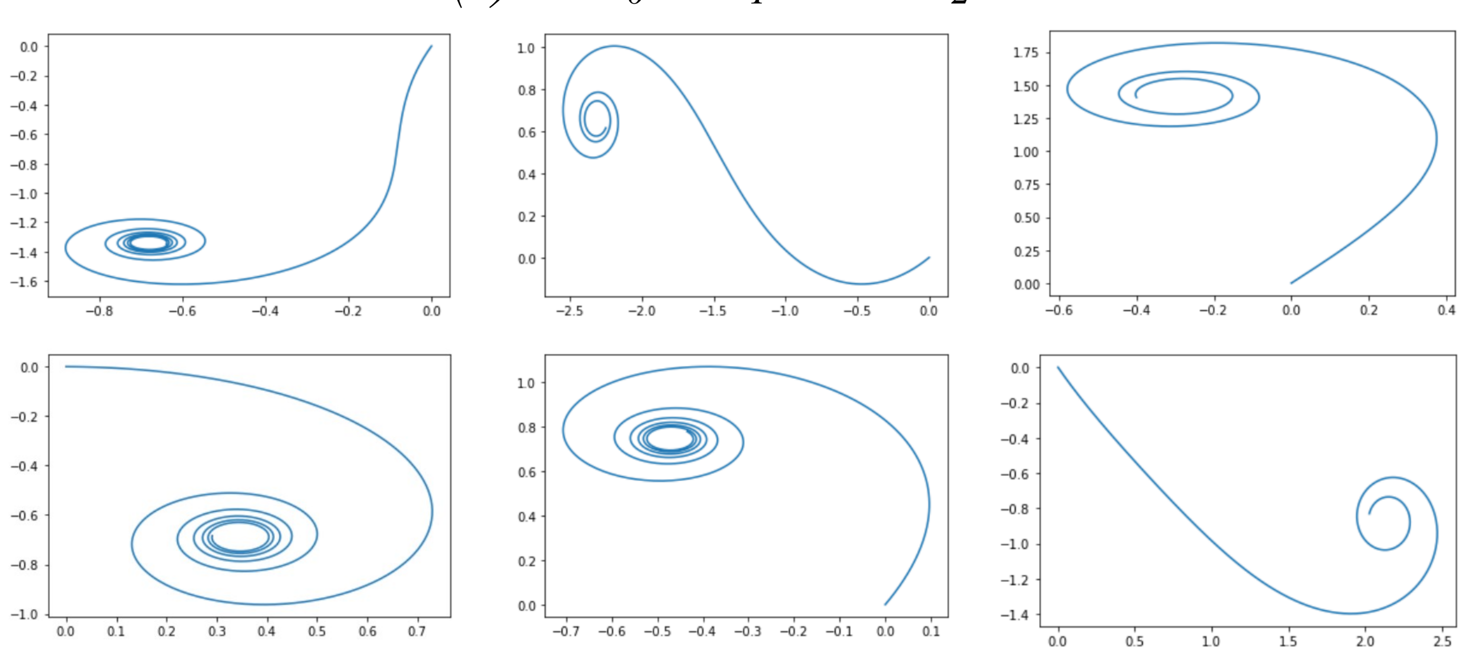
This research project focuses on the spreading of random curves in the differential geometry field which arises in statistical mechanics. It is known from the work of Einstein that random walks are connected to Brownian motion and diffusion. We will examine random curves that are not merely continuous but that are smooth and have prescribed bounds on curvature. We examine the distribution of a finite number of endpoints of such random curves. Using Python, we obtain 2-D histograms, graphs, and charts to research the spreading of random curves. A central goal in statistical mechanics is to describe the large-scale behavior of systems with the distribution of randomly generated data; we compare the distributions of curve endpoints to the Gaussian (normal) distribution.

Process

$$K(s) \to \theta(s) = \int K(s) \to \left(\int \sin\theta(s), \int \cos\theta(s) \right)$$

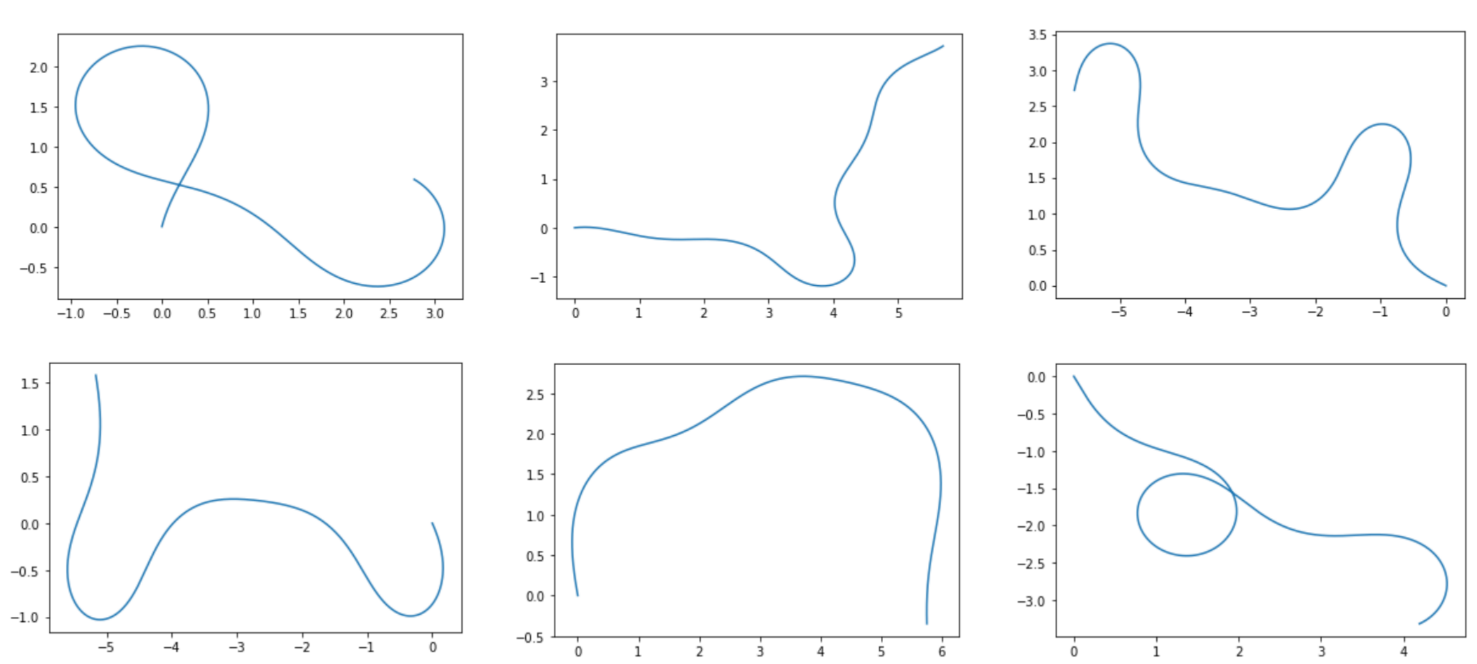
Taylor polynomial curves with random coefficients in interval [-1, 1]

$K(s) = a_0 + a_1 . s + a_2 . s^2$



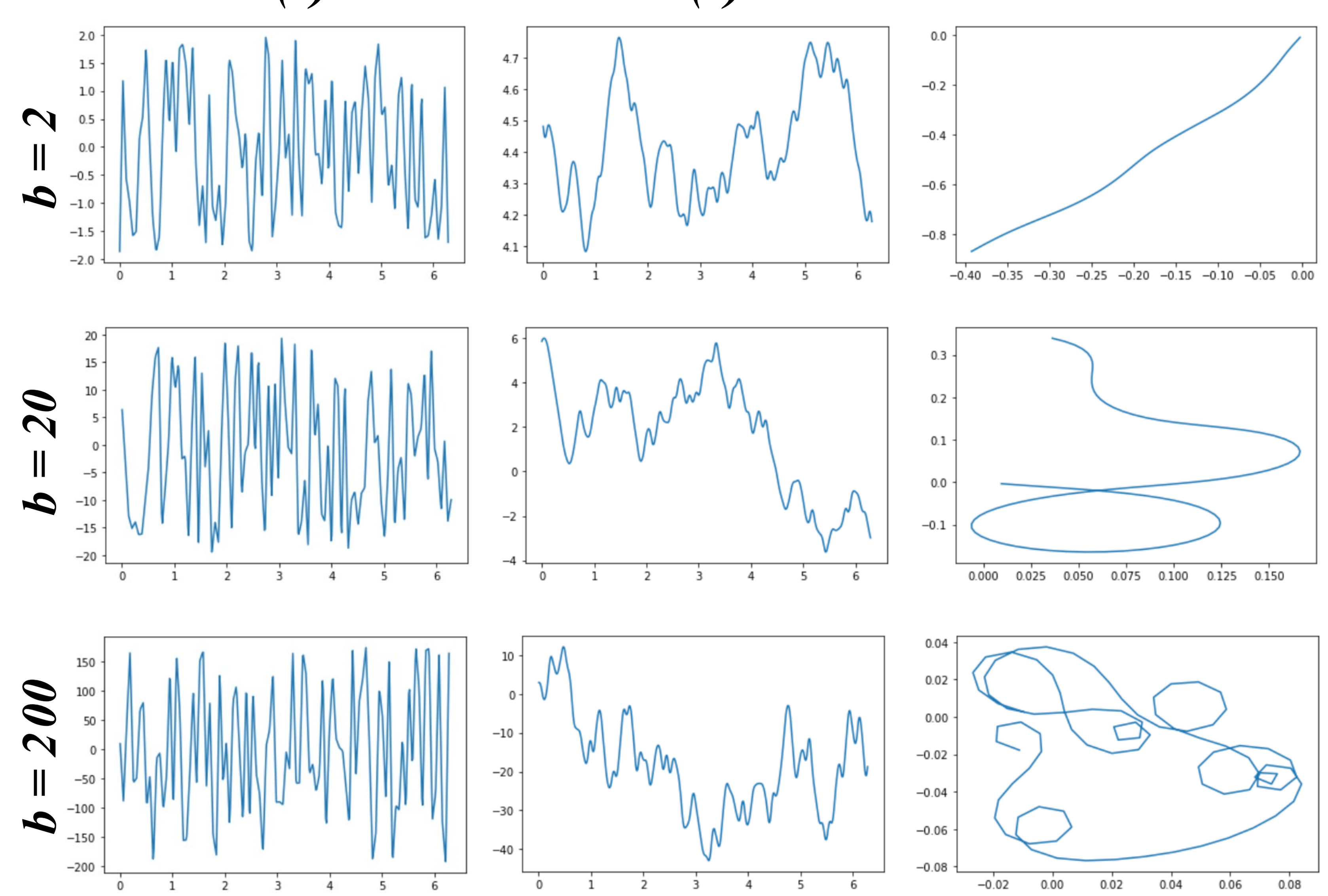
Fourier Series curves with random coefficients in interval [-1,1]

$K(s) = a_0 + a_1 \sin(s) + a_2 \cos(s) + b_1 \sin(2s) + b_2 \cos(2s)$

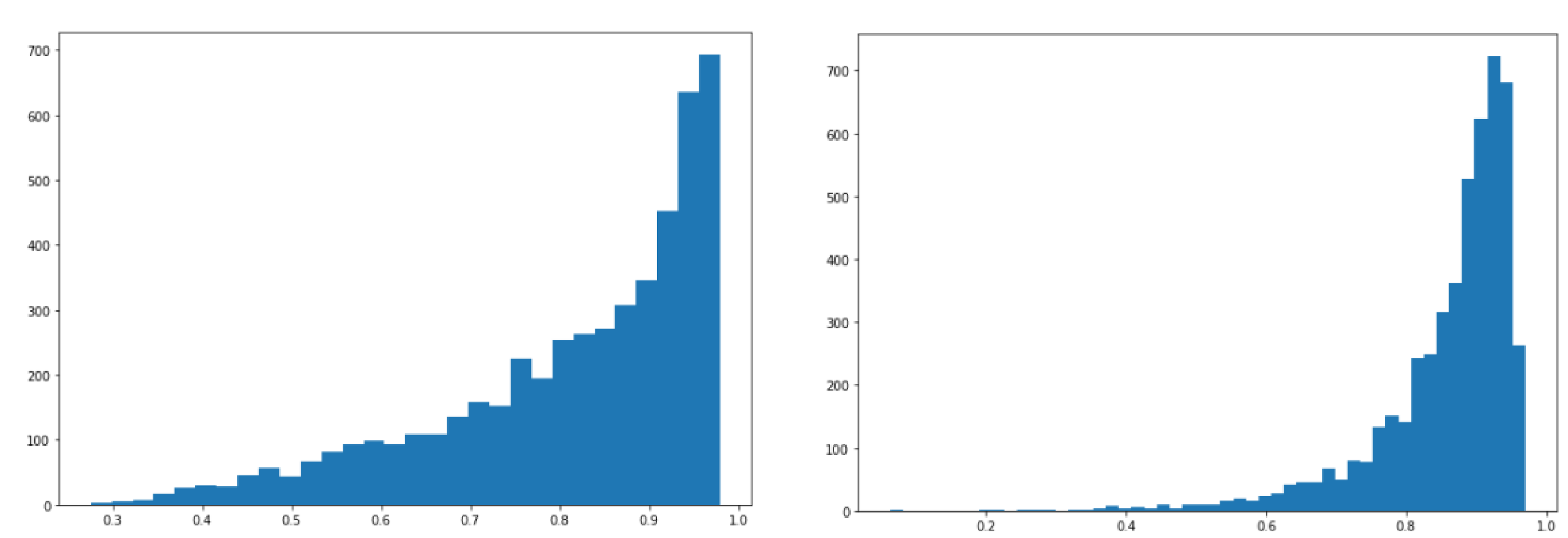


Sample curves generated from any curvatures K(s)

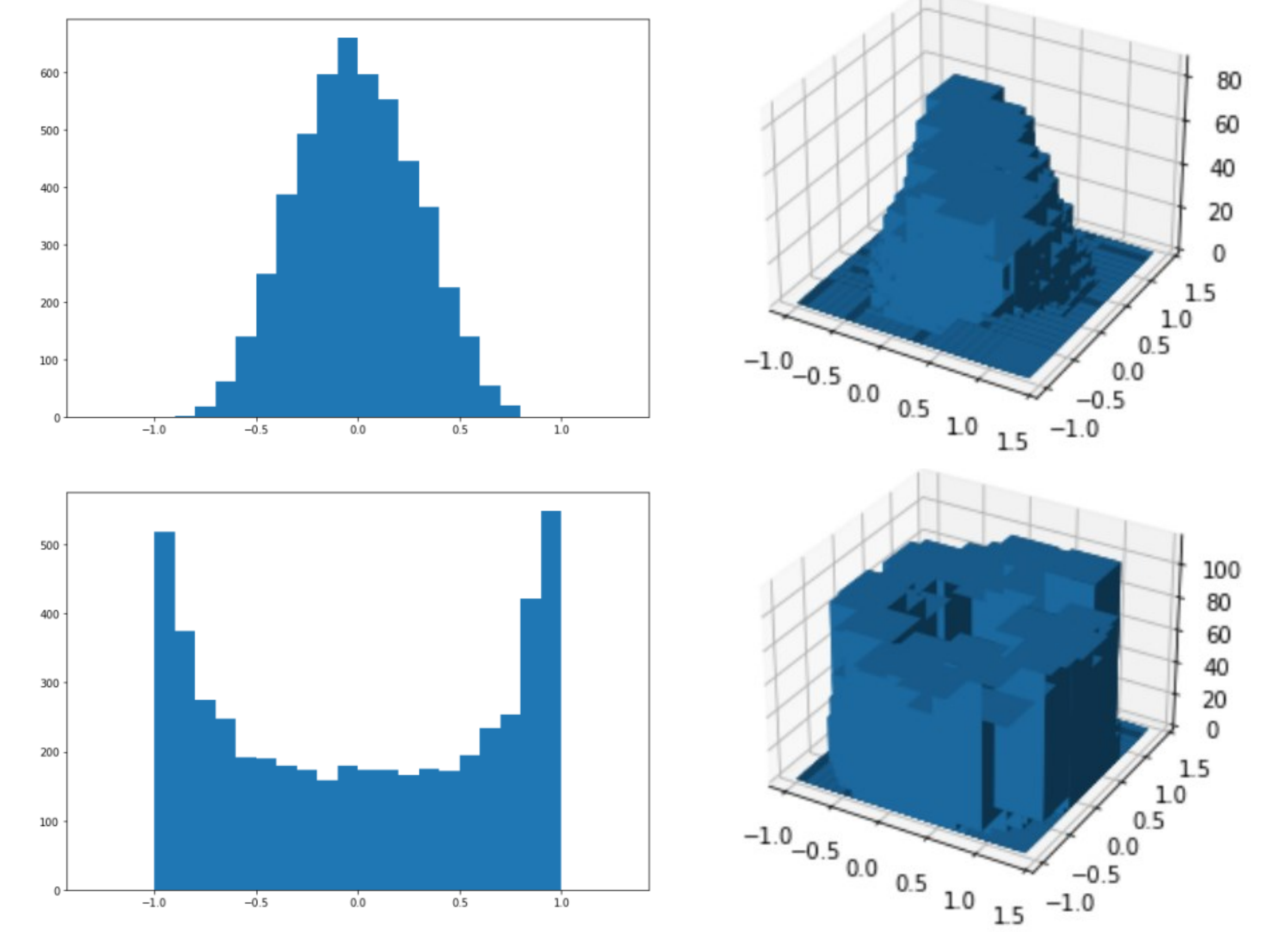
K(s) is a random piecewise-linear bounded function whose values are chosen so that $-b \le K(s) \le b$. *Then, we choose N = number of subdivision of [0, 2π]. There are N random points* $a_1, a_2, \ldots, a_n$ *such that* $-b \le a_j \le b$ *for all j.*



Distribution of radius of random curves



Random endpoints distribution histograms



Gaussian Normal Distribution

The normal distribution is a continuous probability distribution that is symmetrical around its mean, most of the observations cluster around the central peak, and the probabilities for values further away from the mean taper off equally in both directions.