Mathematical modeling of lockdown effectiveness

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Background

- SARS-CoV-2 is a strain of coronavirus that caused the global pandemic that has killed 6.1 million people worldwide.
- SARS-CoV-2 was first identified by the WHO in Wuhan, China in December of 2019 and is the causative agent of a disease called COVID-19.
- COVID-19 affects the respiratory system of an infected individual, and it may cause complications that can lead to death.
- Both the federal and state governments have taken measures to limit the spread of the virus to varying degrees of success.
- The most common measures have been the adoption of social distancing, mask mandates, and lockdowns.
- States have done a good job recording case data which allows for the use of theoretical modeling of COVID-19 transmission on a large scale.

Mathematical Model

We used a Susceptible-Exposed-Infected-Recovered model:

$$\frac{dS}{dt} = -\frac{\beta}{N}SI$$
$$\frac{dE}{dt} = \frac{\beta}{N}SI - kE$$
$$\frac{dI}{dt} = KE - \delta I$$
$$\frac{dR}{dt} = \delta I$$

Each equation represents one group of people and how the number in each group changes over time.

Since the available data was given in cumulative cases, we modeled the cumulative cases as a product of the inverse of the incubation period (k) and the exposed individuals (E).

$$\frac{dC}{dt} = kE,$$

Parameters and Variables

Parameter	Name
β	Infection Rate
k	Incubation Period
δ	Recovery Time
N	State Population
t_{ld}	Time of Lockdown

- With the exception of N, parameters were found by fitting models to data from different states.
- N is fixed to the population of the state.
- We also allow t_{ld} as a fitted parameter.
- This often means that the estimated time of lockdown is on a different day than that of the official lockdown of the government.

Change in Beta

- As time went by during the COVID-19 pandemic states systematically entered states of lockdown and mask mandates were issued.
- These most definitely affected the spread of the virus, and they specifically affected the infection rate of the virus.
- Contact between individuals decreased as many people went outside only when necessary or masks may have helped limit transmission between individuals.
- To account for this in our model, we simulated a change in the β value. We did this in 4 different ways:
  - sudden change from a β_1 value to a lower β_2 on a time of lockdown day (t_{ld}).
  - linear decay from β_1 to β_2 centered around t_{ld}:

$$\beta = \frac{\beta_2 - \beta_1}{t_2 - t_{ld}}t + \frac{(\beta_1 t_2 - \beta_2 t_{ld})}{t_2 - t_{ld}}$$

  - exponential decay from β_1 to β_2 starting at t_{ld}:

$$\beta = \beta_2 + (\beta_1 - \beta_2)e^{\frac{t_{ld} - t}{\tau}}$$

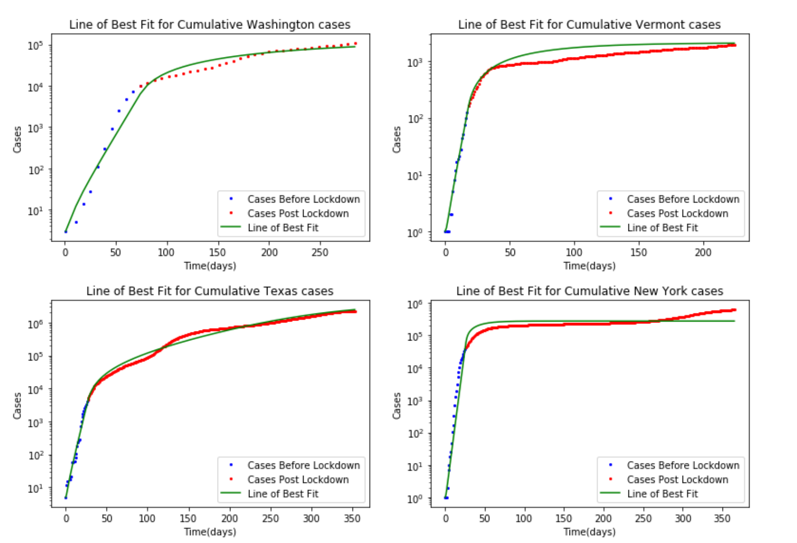
  - logistic decay from β_1 to β_2 centered around t_{ld}:

$$\beta = \beta_2 + \frac{(\beta_1 - \beta_2)}{1 + e^{-\frac{t - t_2}{\tau}}}$$

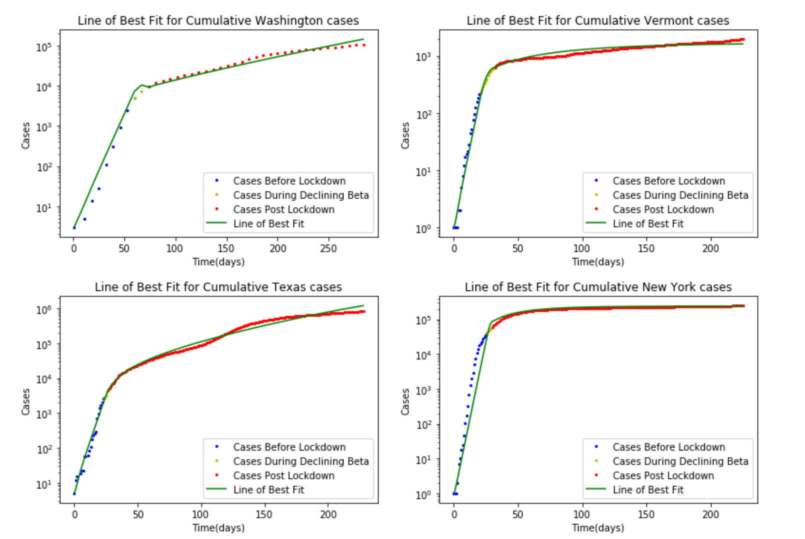
- Best model fits were determined by minimizing the sum of squared residuals (SSR).
- We fit data from 4 different states: Washington, Vermont, Texas, and New York.

Abrupt Change the Infection Rate

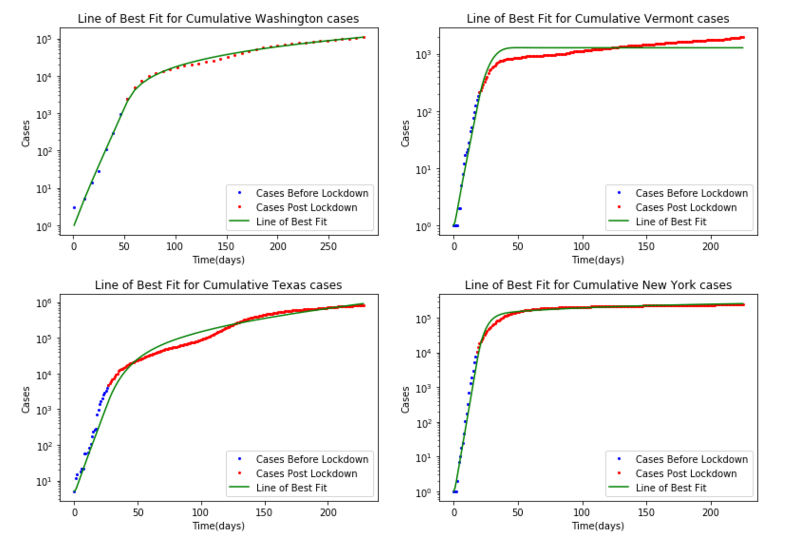
The following graphs show model fits to the cumulative case data in the four states.



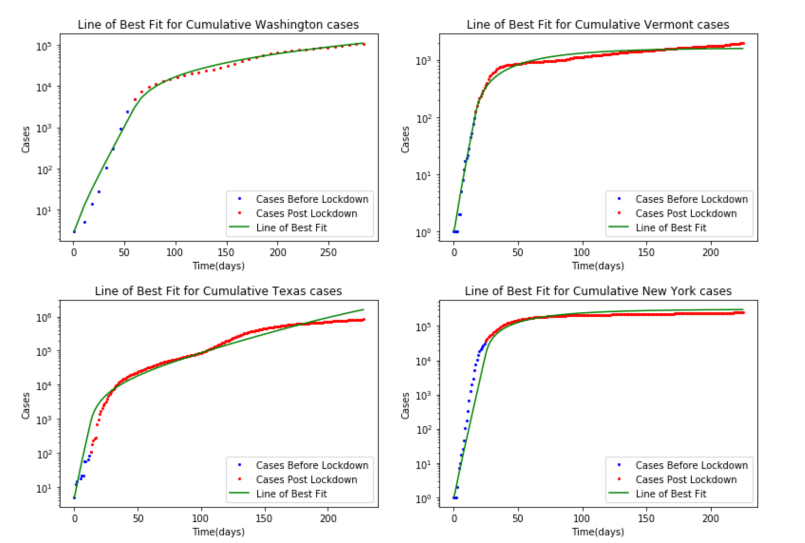
Linear Decay in Infection Rate



Exponential Decay in Infection Rate

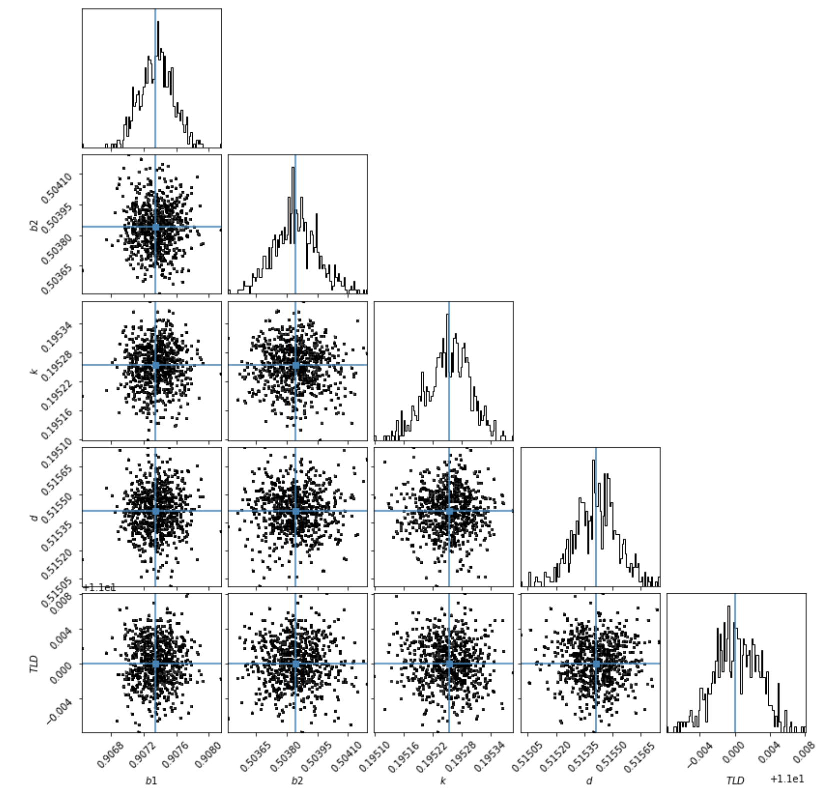


Logistic Change in Infection Rate



MCMC

- After fitting each model to the data, we then need to find possible distributions for each parameter.
- We used Monte Carlo Markov Chain (MCMC) to find a range of where each parameter most likely is by using our optimized parameters as a base.
- A sample MCMC result is shown below.



Model Comparison

We use Akaike's Information Criterion (AIC) to determine the best model for changing β.

Model	Washington	Vermont	Texas	New York
Abrupt	-126	**-1240**	-934	-594
Linear	-147	-1200	**-1060**	-685
Exp.	**-176**	-947	-762	**-1040**
Log.	-165	-1210	-696	-907
Best	Exp.	Abrupt	Linear	Exp.

Conclusions

- Effective date of lockdown was different from that of the actual lockdown.
- Other factors like local responses or behavior may have contributed to this difference.
- Preventive measures had considerable impact on the infection rate of SARS-CoV-2.