Modeling the effect of multiple vaccines on the spread of SARS-CoV-2

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Background

- In late 2019, SARS-CoV-2 emerged from Wuhan, China and has proceeded to rapidly spread across the world.
- In response to this pandemic, several different vaccines have been introduced to combat spread of the virus.
- Since the virus is capable of mutating to escape the protection of a vaccine, using multiple vaccines is believed to help reduce the spread.
- We use mathematical models to simulate the effect of using multiple vaccines on the virus.

Single Vaccine Model

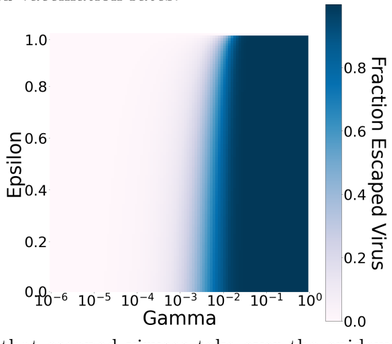
We used a mathematical model simulating the spread of the virus with a single vaccine present. In this model, people who recover from infection are assumed to have immunity to the virus afterwards.

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= \delta N - \frac{\beta_w}{N} I_w S - \frac{\beta_e}{N} I_e S - \gamma S - \delta S \\
\frac{\mathrm{d}I_w}{\mathrm{d}t} &= \frac{\beta_w}{N} I_w S - (\delta + \alpha) I_w \\
\frac{\mathrm{d}I_e}{\mathrm{d}t} &= \frac{\beta_e}{N} I_e S + \frac{\beta_e}{N} I_e V + (1-\epsilon)\frac{\beta_w}{N} I_w V - (\delta + \alpha) I_e \\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \gamma S - \frac{\beta_e}{N} I_e V - (1-\epsilon)\frac{\beta_w}{N} I_w V - \delta V \\
\frac{\mathrm{d}R}{\mathrm{d}t} &= \alpha(I_w + I_e) - \delta R.
\end{aligned}$$

- We assume that any breakthrough infection after vaccination is an escaped mutant virus.
- SARS-CoV-2 parameters are taken from literature.
- While the model allows for different infection rates for wild-type and escaped virus, we assume the same infection rate.

Escaping the Vaccine

The heatmap displays the fraction of viruses that escape the vaccine throughout the simulation given varying vaccine efficacy and vaccination rates.



We find that escaped viruses take over the epidemic unless vaccination rates are low and vaccine efficacy is high.

Fixed Points

- This model had two fixed points. The first point was a Disease Free Equilibrium point where all infections went to 0. The other point was an endemic point where the system approached a constant amount of escaped virus infected people.
- The basic reproduction number for this model is the maximum of:

$$\frac{\beta_w \delta}{(\alpha+\delta)(\delta+\gamma)} \qquad \frac{\beta_e}{\alpha+\delta}$$

Waning Immunity

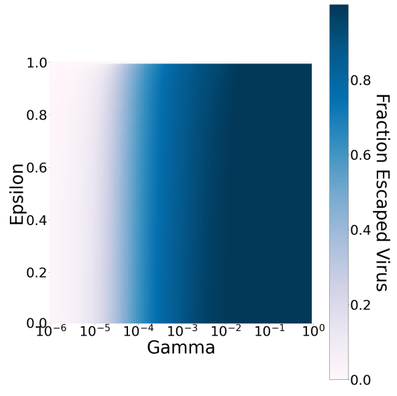
We improved upon the single vaccine model by implementing waning immunity into the simulation. Post infection, people will only be safe from the virus for a short period of time before becoming susceptible again.

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= \delta(N-D) - \frac{\beta_w}{N} I_w S - \frac{\beta_e}{N}(I_e + I_{eu})S - \gamma S - \delta S + \omega R \\
\frac{\mathrm{d}I_w}{\mathrm{d}t} &= \frac{\beta_w}{N} I_w S - (\delta + \alpha + \delta_v) I_w \\
\frac{\mathrm{d}I_{eu}}{\mathrm{d}t} &= \frac{\beta_e}{N}(I_e + I_{eu})S - (\delta + \alpha + \delta_v) I_{eu} \\
\frac{\mathrm{d}I_e}{\mathrm{d}t} &= \frac{\beta_e}{N}(I_e + I_{eu})V + (1-\epsilon)\frac{\beta_w}{N} I_w V - (\delta + \alpha + \delta_v) I_e \\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \gamma(S+R) - \frac{\beta_e}{N}(I_e + I_{eu})V - (1-\epsilon)\frac{\beta_w}{N} I_w V - \delta V + \omega R_v \\
\frac{\mathrm{d}R}{\mathrm{d}t} &= \alpha(I_w + I_{eu}) - \gamma R - \delta R - \omega R \\
\frac{\mathrm{d}R_v}{\mathrm{d}t} &= \alpha I_e - \delta R_v - \omega R_v \\
\frac{\mathrm{d}D}{\mathrm{d}t} &= \delta_v(I_w + I_{eu} + I_e)
\end{aligned}$$

Vaccinated people can lose immunity to the escaped strain and unvaccinated people lose immunity to both the escaped and wild-type virus

Easier Escape

Waning immunity makes it easier for the virus to escape the vaccine.



Fixed Points

- This model had two Disease Free Equilibrium Fixed Points. In one point the vaccinated population reaches a finite value while in the other the vaccinated population is variable.
- The basic reproduction number for this model is the maximum of:

$$\frac{\beta_e \delta V^*}{\gamma N(\alpha+\delta+\delta_v)} \qquad \frac{\beta_w \delta V^*}{\gamma N(\alpha+\delta+\delta_v)} \qquad \frac{\beta_e V^*}{N(\alpha+\delta+\delta_v)}$$

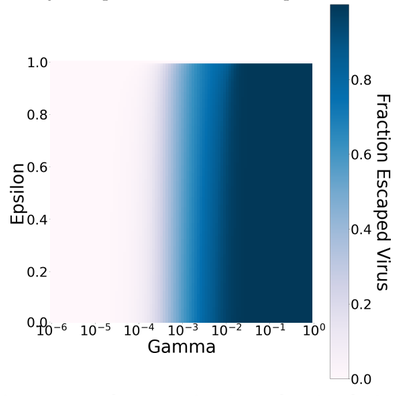
Two Vaccine Model

We developed a model to determine the effects of having 2 vaccines on the spread of the virus. In this model, waning immunity is also included so it is possible to become reinfected.

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= \delta(N-D) - \frac{\beta_0}{N} I_0 S - \sum_i \frac{\beta_i}{N} \sum_{j \neq i} I_i^{(j)} S - \frac{\beta_e}{N} \sum_j I_e^{(j)} S \\
&\quad - \sum_i \gamma_i S - \delta S + \omega R_0 \\
\frac{\mathrm{d}I_0}{\mathrm{d}t} &= \frac{\beta_0}{N} I_0 S - (\delta + \alpha + \delta_v) I_0 \\
\frac{\mathrm{d}I_i^{(j)}}{\mathrm{d}t} &= \frac{\beta_i}{N} \sum_k I_i^{(k)} V_i + (1-\epsilon_j)\frac{\beta_0}{N} I_0 V_i - (\delta + \alpha + \delta_v) I_i^{(j)} \\
\frac{\mathrm{d}I_e^{(j)}}{\mathrm{d}t} &= \frac{\beta_e}{N} \sum_k I_e^{(k)} V_j + \sum_k (1-\epsilon_k)\frac{\beta_j}{N} \sum_l I_j^{(l)} V_j - (\delta + \alpha + \delta_v) I_e^{(j)} \\
\frac{\mathrm{d}V_j}{\mathrm{d}t} &= \gamma_j(S + R_0) - \frac{\beta_j}{N} \sum_k I_j^{(k)} V_j - (1-\epsilon_j)\frac{\beta_0}{N} I_0 V_j \\
&\quad - \sum_k (1-\epsilon_k)\frac{\beta_j}{N} \sum_l I_j^{(l)} V_j - \frac{\beta_e}{N} \sum_k I_e^{(k)} V_j - \delta V_j + \omega R_j \\
\frac{\mathrm{d}R_0}{\mathrm{d}t} &= \alpha \sum_i I_i^{(0)} - \sum_i \gamma_i R_0 - \delta R_0 - \omega R_0 \\
\frac{\mathrm{d}R_j}{\mathrm{d}t} &= \alpha(I_1^{(j)} + I_e^{(j)}) - \delta R_j - \omega R_j \\
\frac{\mathrm{d}D}{\mathrm{d}t} &= \delta_v \sum_i \sum_j I_j^{(i)}
\end{aligned}$$

Harder Escape

We assume that people only receive one vaccine, but for simplicity. A fully escaped virus must escape both vaccines.



Introducing a secondary vaccine into the simulation greatly increases the proportion of situations in which the virus is unable to mutate out of the vaccines.

Fixed Points

- This model has a single fixed point. This point is a Disease Free Equilibrium Point.
- The basic reproduction number for this model is the maximum of:

$$\frac{\beta_0 \delta(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)} \quad \frac{\beta_1 \delta(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)} \quad \frac{b2\delta(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)}$$
$$\frac{\beta_e \delta(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)} \quad \frac{\beta_1 \gamma_1(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)} \quad \frac{\beta_e \gamma_1(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)}$$
$$\frac{\beta_2 \gamma_2(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)} \quad \frac{\beta_e \gamma_2(N-D)}{N(\alpha+\delta+\delta_v)(\delta+\gamma_1+\gamma_2)}$$

Conclusions

- With a single vaccine, only low vaccination rate and high vaccine efficacy prevent the virus from escaping the vaccine.
- Waning immunity makes it easier for the virus to escape.
- Increasing the number of vaccines reduces the probability that a virus will escape all vaccines.
- This means that viral spread can be better contained with more vaccines as viral mutations becomes less effective.

Future Directions

- Going forward, we hope to create a general model to simulate the spread of SARS-CoV-2 with n vaccines.
- We plan to use nonlinear analysis to determine whether there is a minimum number of vaccines that could completely stop the virus from escaping vaccine protection.
- We plan to incorporate asymptomatic infections since this is a primary driver of spread of SARS-CoV-2.

SciCom Let's Talk Science

In this project, we sought to find how the number of vaccines available in a population affects the spread of Covid-19. To do this, we developed mathematical models to simulate the spread of the virus in three situations: a single vaccine, a single vaccine with waning immunity, and two vaccines. We found that as we increase the number of vaccines, it becomes harder for the virus to spread. This means that a greater number of vaccines results in easier prevention of large-scale infections.