Mathematical modeling of lockdown effectiveness

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Background

- SARS-CoV-2 is a strain of coronavirus that caused the global pandemic that has killed 6.8 million people worldwide.
- COVID-19 affects the respiratory system of an infected individual, and it may cause complications that can lead to death.
- Before a vaccine or treatment was available, the most common preventive measures were the adoption of social distancing, mask mandates, and lockdowns.
- States have done a good job recording case data, which allows for the use of theoretical modeling of SARS-CoV-2 transmission on a large scale.
- We used this data to examine how to model and measure the effectiveness of non-pharmaceutical interventions.

Mathematical Model

We used a Susceptible-Exposed-Infected-Recovered model:

$$\frac{dS}{dt} = -\frac{\beta}{N}SI$$
$$\frac{dE}{dt} = \frac{\beta}{N}SI - kE$$
$$\frac{dI}{dt} = KE - \delta I$$
$$\frac{dR}{dt} = \delta I$$

Each equation represents one group of people and how the number in each group changes over time.

$$\text{Susceptible} \xrightarrow{\beta} \text{Exposed} \xrightarrow{k} \text{Infected} \xrightarrow{\delta} \text{Recovered}$$

Since the available data was given in cumulative cases, we modeled the cumulative cases as a product of the inverse of the incubation period (k) and the exposed individuals (E).

$$\frac{dC}{dt} = kE$$

Parameters and Variables

Parameter	Name
β	Infection Rate
k	Incubation Period
δ	Recovery Time
N	State Population
t_{ld}	Time of Lockdown

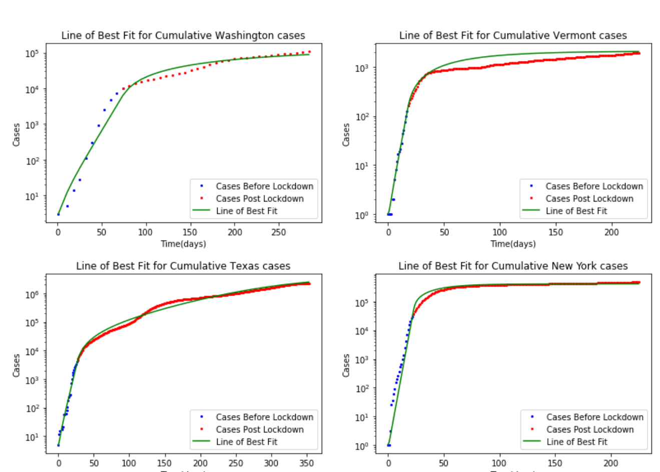
- With the exception of N, parameters were found by fitting models to data from different states.
- N is fixed to the population of the state.
- We also allow t_{ld} as a fitted parameter.

Change in Beta

- As time went by during the COVID-19 pandemic states systematically entered states of lockdown and mask mandates were issued.
- These most definitely affected the spread of the virus, and they specifically affected the infection rate of the virus as contact between individuals decreased due to preventative measures.
- To account for this in our model, we simulated a change in the β value. We did this in 4 different ways:
  - sudden change from a β_1 value to a lower β_2 on a time of lockdown day (t_{ld}).
  - linear decay from β_1 to β_2 centered around t_{ld}:
    $$\beta = \frac{\beta_2-\beta_1}{t_2-t_{ld}}t + \frac{(\beta_1 t_2-\beta_2 t_{ld})}{t_2-t_{ld}}$$
  - exponential decay from β_1 to β_2 starting at t_{ld}:
    $$\beta = \beta_2 + (\beta_1 - \beta_2)e^{\frac{t_{ld}-t}{\tau}}$$
  - logistic decay from β_1 to β_2 centered around t_{ld}:
    $$\beta = \beta_2 + \frac{(\beta_1-\beta_2)}{1+e^{-\frac{t-t_2}{\tau}}}$$
- Best model fits were determined by minimizing the sum of squared residuals (SSR) from 4 different states: Washington, Vermont, Texas, and New York.

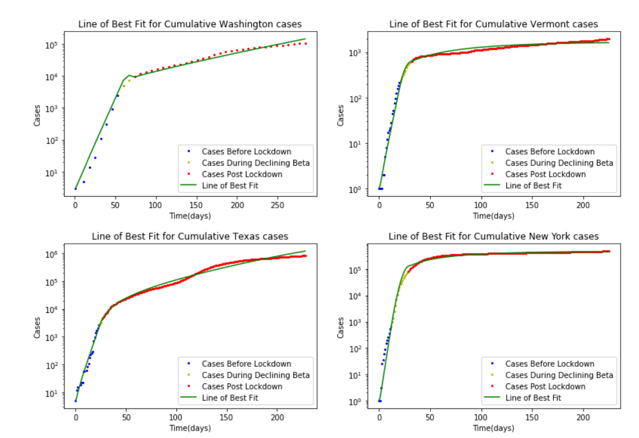
Abrupt Change of β

We show model fits to the cumulative case data in the four states under the assumption of an abrupt change in the infection rate.



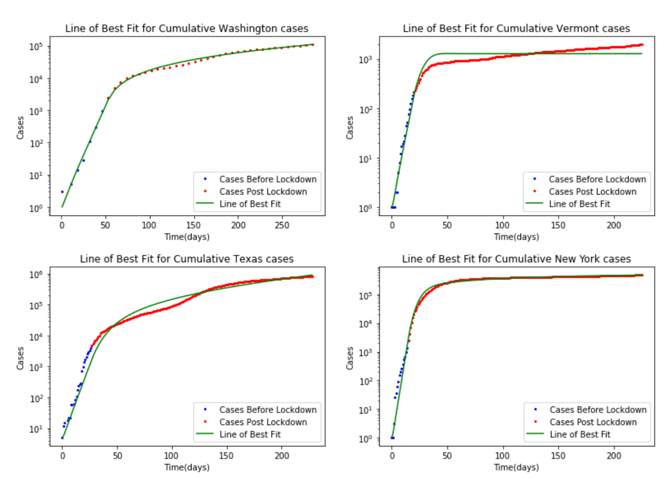
Linear Decay of β

Below are model fits under the assumption of a linear change in the infection rate.



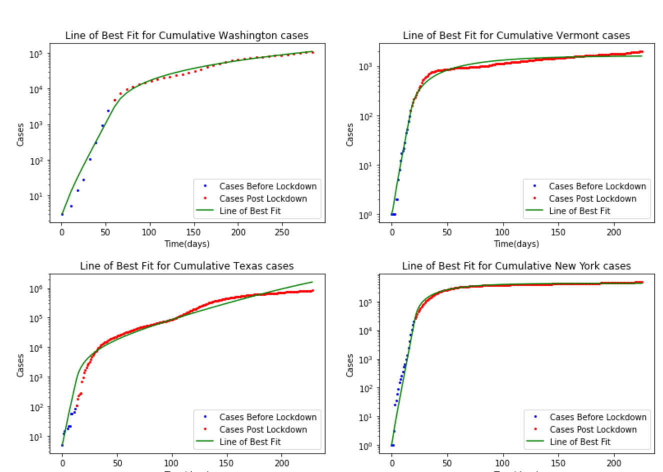
Exponential Decay of β

Below are model fits under the assumption of an exponential change in the infection rate.



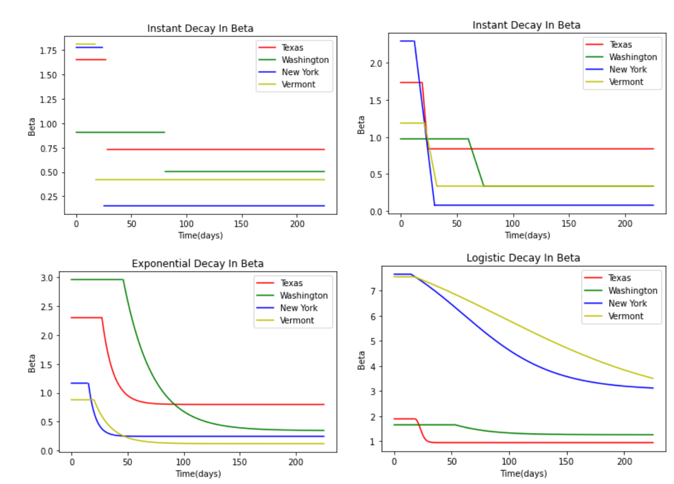
Logistic Decay of β

Below are model fits under the assumption of a logistic change in the infection rate.



Beta Change Graphs

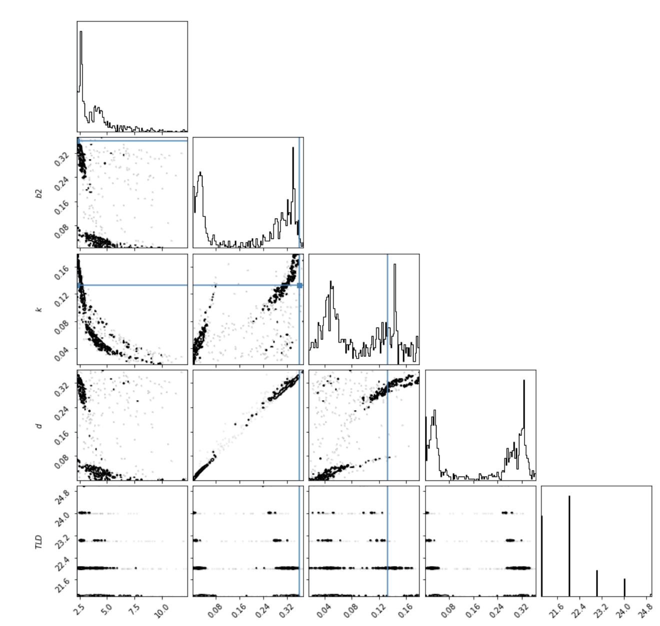
Below are the visualizations of each model decrease in β for each respective state.



- Logistic decay leads to very high initial values of transmission rate for some states.
- All models predict a relatively late change in transmission rate for the state of Washington.

Bootstrapping

For each model, each parameter was estimated with a 95 percent confidence level using bootstrapping. An example plot can be seen below.



While the parameter estimates have reasonably narrow distributions, we see that there are correlations between some of the parameters.

Model Comparison

We use Akaike's Information Criterion (AIC) to determine the best model for changing β.

$$AIC = n\ln\left(\frac{SSR}{n}\right) + \frac{2(K+1)n}{n-K-2}$$

AIC penalizes for extra parameters.

Model	Washington	Vermont	Texas	New York
Instantaneous	-126	**-1240**	-934	-769
Linear	-147	-1200	**-1060**	-924
Exponential	**-176**	-947	-762	-918
Logistic	-165	-1210	-696	**−933**

Conclusions

- Effective date of the lockdown was different from that of the actual lockdown date.
- Other factors like local responses or behavior might have contributed to this difference.
- Preventive measures had a considerable impact on the infection rate of SARS-CoV-2.
- Demonstrates the non-constant nature of the infection rate during the SARS-CoV-2 pandemic as a result of preventative measures.

Limitations

- More accurate and complex models could be used for better fits.
- Small sample size limited conclusions.
- Could not differentiate between preventative measures and which ones caused the largest change.

SciCom Let's Talk Science

This study analyzed the effect preventative measures had on SARS-CoV-2 transmission rates within the U.S. We used a mathematical model with a variable transmission rate and fit SARS-CoV-2 case data from four states to it. We tested four models for the change in transmission rate: instant, linear, exponential, and logistic. After comparing models between the four states, there was no clear best model for the change in transmission due to preventive measures. These results suggest that regional differences like behavior, socioeconomic status, and exact preventative measures enforced could be responsible for the disparity in how the transmission rate decayed.