Computations about Cheeger's Constant

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Cheeger's Constant

Cheeger's constant is a measure that tells how connected a graph is.
It's calculated with the following formula: [2]

$$h(G) := \min\left\{\frac{|\partial A|}{|A|} \ : \ A \subseteq V(G), 0 < |A| \leq \tfrac{1}{2}|V(G)|\right\}$$

h(G) = 1

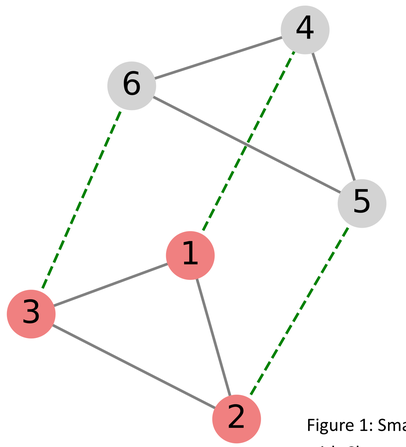


Figure 1: Small graph with Cheeger's constant

Applications

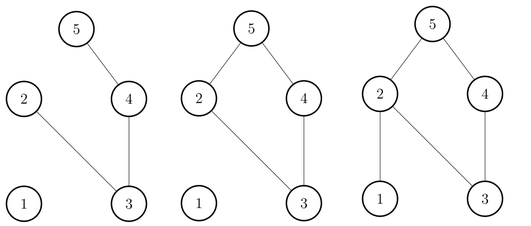
Network Analysis - Checking for less connected portions of network systems can assist in identifying bottlenecks and inform the user where redundancies are required.

Parallel Computing - This constant can help in designing parallel systems to lower communication overhead needed within the system.

Urban Planning - Similar to network analysis, looking for congestion spots where traffic is forced through one point will benefit from this constant.

Graph Generation

Low Density Graph Generation

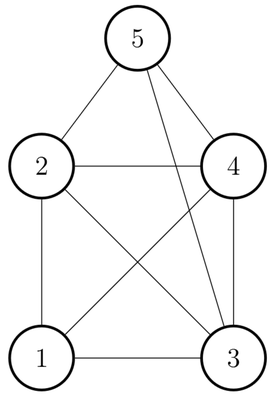


Initial Graph with 0.3 edge odds

Select a random node and add an edge

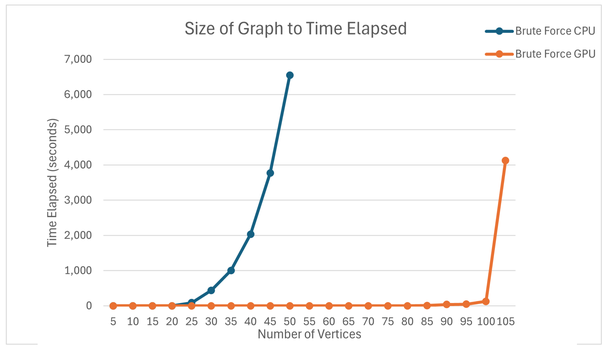
Repeat until graph is completed

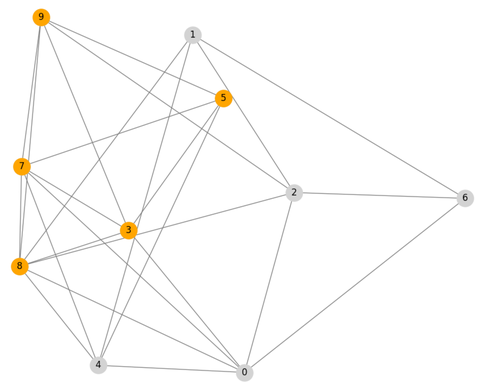
High Density Graph Generation

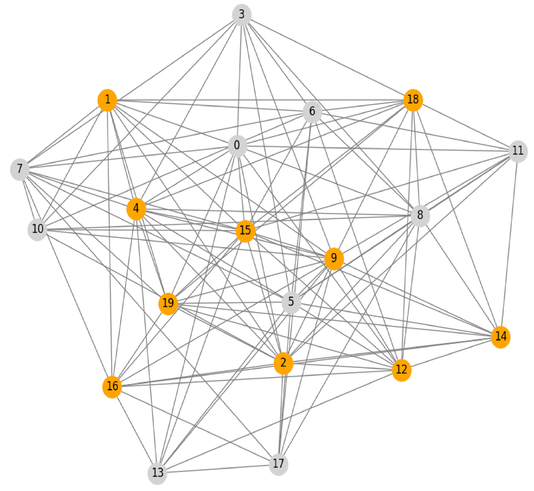


This will be created by using an edge odd of 0.7

Calculations







Future Research

1. Investigating the effects of submatrix computation for larger graphs. This may help prevent the spike in computation time with GPU calculations, as it might help with the transfer of data to be more efficient.
2. Analyze the performance in a heuristics model, especially in the aspects of machine learning. See how the tradeoffs of accuracy with time efficiency changes with this approach.

Citations

*[1]*By BagLuke - Own work, CC0, *https://commons.wikimedia.org/w/index.php?curid=161321096*

[2]*Mohar, Bojan (1989). "Isoperimetric numbers of graphs". Journal of Combinatorial Theory, Series B. **47** (3): 274–291. doi:10.1016/0095-8956(89)90029-4.*

Acknowledgement

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