Vaccine optimization using a simplified epidemiological model

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Background

- During pandemics, vaccine supply is often limited early, so allocation strategy matters.
- Severe outcomes are not evenly distributed across the population.
- Mathematical models can test vaccination strategies quickly and transparently.
- We developed a simplified compartmental model stratified by hospitalization risk and infection susceptibility.
- Goal: identify prioritization strategies that minimize infections, hospital burden, and deaths.

Model structure

The population is divided into four susceptible groups:

$$S_{ll},\ S_{lh},\ S_{hl},\ S_{hh}$$

representing low/high hospitalization risk and low/high infection susceptibility.
Additional compartments:

$$I_l,\ I_h,\ H,\ R$$

Vaccination occurs at group-specific rates:

$$\gamma_{ll},\ \gamma_{lh},\ \gamma_{hl},\ \gamma_{hh}$$

Key idea: compare which group should receive vaccines first when supply is limited.

Mathematical model

$$\frac{dS_{ll}}{dt} = -\frac{\beta_l}{N}S_{ll}(I_l+I_h) - \gamma_{ll}S_{ll}$$
$$\frac{dS_{lh}}{dt} = -\frac{\beta_l}{N}S_{lh}(I_l+I_h) - \gamma_{lh}S_{lh}$$
$$\frac{dS_{hl}}{dt} = -\frac{\beta_h}{N}S_{hl}(I_l+I_h) - \gamma_{hl}S_{hl}$$
$$\frac{dS_{hh}}{dt} = -\frac{\beta_h}{N}S_{hh}(I_l+I_h) - \gamma_{hh}S_{hh}$$
$$\frac{dI_l}{dt} = \frac{\beta_l}{N}S_{ll}(I_l+I_h) + \frac{\beta_h}{N}S_{hl}(I_l+I_h) - \alpha_l I_l$$
$$\frac{dI_h}{dt} = \frac{\beta_l}{N}S_{lh}(I_l+I_h) + \frac{\beta_h}{N}S_{hh}(I_l+I_h) - \alpha_h I_h - \rho I_h - \delta I_h$$
$$\frac{dH}{dt} = \rho I_h - \alpha_h H - \delta H$$
$$\frac{dR}{dt} = \sum \gamma_i S_i + \alpha_l I_l + \alpha_h (I_h + H) + \delta(I_h + H)$$

Parameters

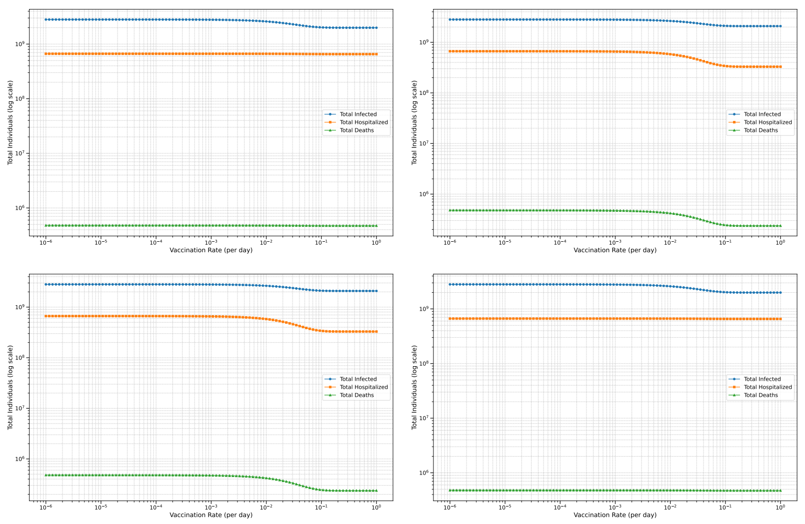
- β_l, β_h: transmission rates
- α_l, α_h: recovery rates
- ρ: hospitalization flow
- δ: mortality rate

Simulations

- ODE system solved in Python using `scipy.integrate.odeint`.
- Initial population evenly divided among the four susceptible groups.
- Three analyses were performed:
  - single-group vaccination scans
  - sequential vaccination with fixed time windows
  - PRCC sensitivity analysis
- Outcomes recorded:
  - infected person-days
  - hospitalized person-days
  - modeled deaths

Single-group vaccination

We varied the vaccination rate for one group at a time over a wide range of values.



Main result

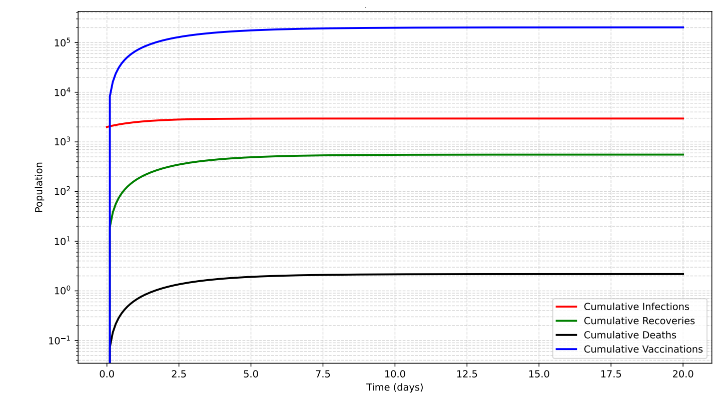
- Vaccinating the high-risk, high-susceptibility group produces the largest reductions in hospitalizations and deaths.
- Vaccinating low-risk groups has much smaller effects on severe outcomes.
- Clear threshold behavior appears near intermediate vaccination rates.

Sequential vaccination strategy

We tested a fixed-window priority strategy:

$$S_{hh} \rightarrow S_{hl} \rightarrow S_{lh} \rightarrow S_{ll}$$

with 5-day windows over a 20-day horizon.

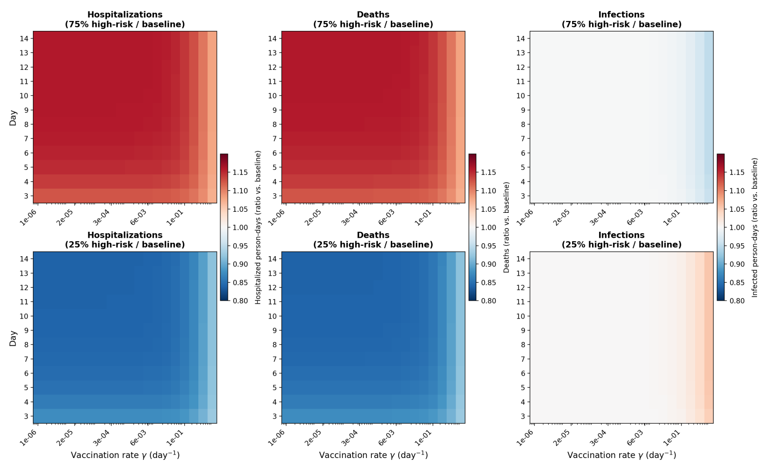


Results

- Rapid early vaccination strongly suppresses transmission.
- Cumulative infections and deaths flatten quickly after high-risk groups are targeted first.
- Front-loading vaccines to the highest-risk group is highly effective over short horizons.

Population distribution effects

We compared outcome ratios against a 50% high-risk baseline for different population mixes and priority orders.



Interpretation

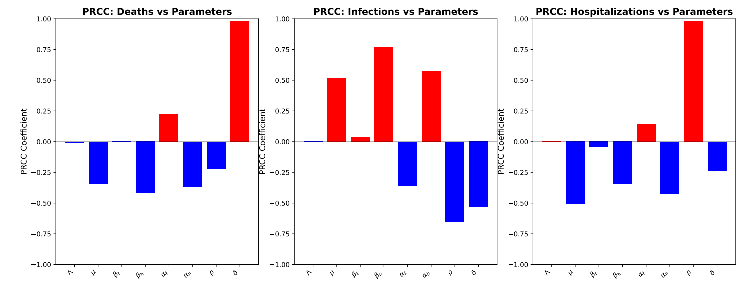
- Ratio < 1: improved outcome relative to baseline
- Ratio > 1: worse outcome relative to baseline

Takeaway

- Priority order matters most when the population is high-risk heavy.
- Best performance occurs when high-risk, high-susceptibility groups are vaccinated first.

Sensitivity analysis

We used partial rank correlation coefficients (PRCC) to determine which parameters most strongly influence outcomes.



Key findings

- Mortality rate δ is the dominant driver of deaths.
- Hospitalization flow ρ is the strongest driver of hospital burden.
- Severe outcomes are influenced more by clinical progression parameters than by low-risk transmission parameters.

Conclusions

- Vaccinating high-risk, high-susceptibility groups first gives the greatest reduction in severe outcomes.
- Simple sequential policies can perform well while remaining operationally easy to implement.
- Population composition influences how strongly prioritization affects outcomes.
- Mortality and hospitalization processes remain critical even when vaccination is available.

Future directions

- Include age structure and comorbidities
- Model waning immunity and booster doses
- Add operational constraints such as delivery capacity
- Explore robust decision-making under deep uncertainty
- Compare against more detailed agent-based or hybrid models

References

References